Ground state and collective modes of dipolar BECs

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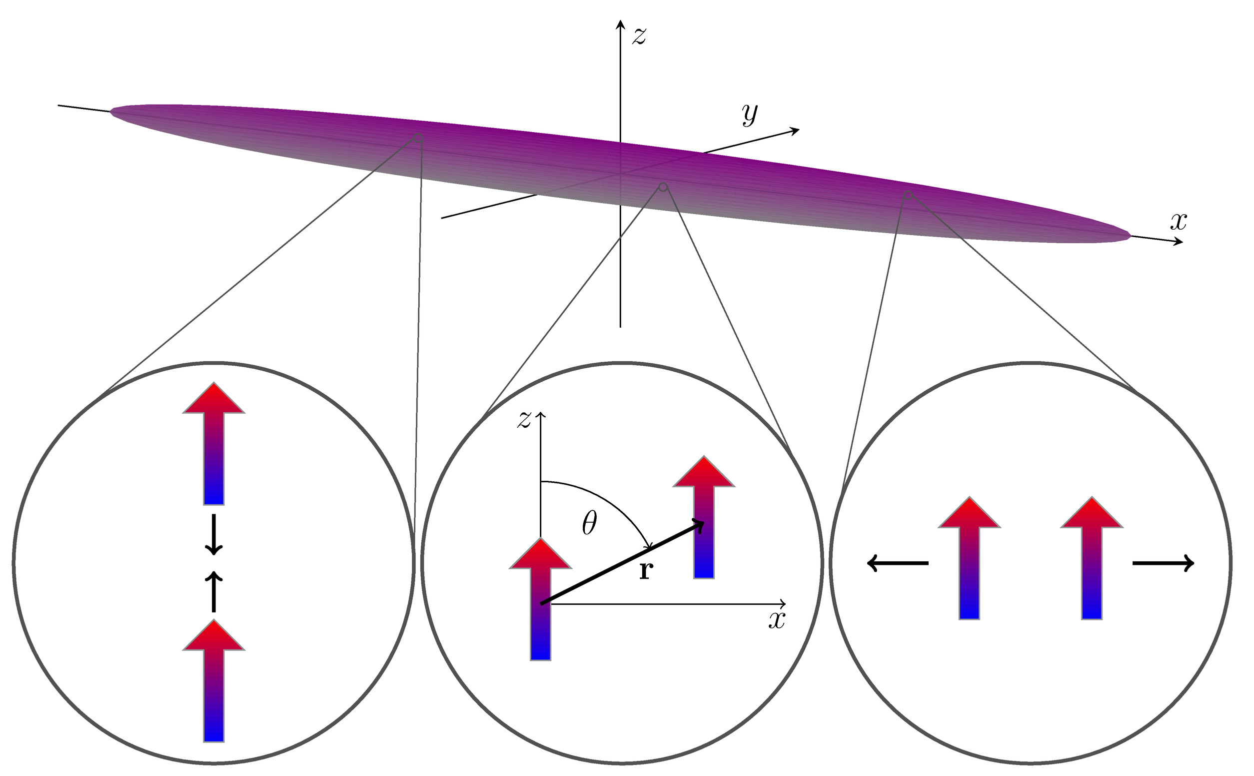
Abstract

We study the effects of the dipole-dipole interaction on the ground state and collective modes of quasi-one-dimensional dipolar Bose-Einstein condensates of the atomic gas of erbium ^{168}Er. Through extensive numerical simulations and detailed variational treatment, we analyze the dependence of condensate widths on the dipole-dipole interaction strength, as well as the interaction-induced frequency shifts of collective oscillation modes. Furthermore, we show that the Gaussian variational approach gives a good qualitative description of the system's ground state, and an excellent quantitative description of the condensates' low-lying excitation modes.

Parameters of the system

- Condensate is confined into a cigar-shaped harmonic trap [1] $\omega_x = 7 \times 2\pi$ Hz, $\omega_y = \omega_z = 160.5 \times 2\pi$ Hz
- All simulations [2,3] and calculations are performed with the same number of atoms $N = 10^4$
- Spatial discretization mesh $N_x = N_y = N_z = 500$, with different spacings $\Delta x = 0.5$, $\Delta y = \Delta z = 0.1$ corresponds to the simulation box of the volume $250 \times 50 \times 50\,\mu\text{m}^3$
- Time discretization $N_t = 10^5$, with time step $\Delta t = 10^{-3}$ corresponds to the simulation of the evolution 1000 ms
- Whenever one of the interaction strengths has a fixed value, we use the data for ^{168}Er: $a_s = 100\,a_0$ and $a_{\text{dd}} = 67\,a_0$



Variational description of the dipolar Bose gas in a trap

- Gross-Pitaevskii equation for dipolar BECs has two types of nonlinearities due to the two types of interactions: the contact and the dipole-dipole interaction

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r},t) + \frac{4\pi\hbar^2 a_s}{m}|\psi(\mathbf{r},t)|^2 + \int d\mathbf{r}'\,\psi^*(\mathbf{r}',t)U_{\text{dd}}(\mathbf{r}-\mathbf{r}')\psi(\mathbf{r}',t)\right]\psi(\mathbf{r},t)\,, \qquad U(\mathbf{r},t) = \frac{m}{2}(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2)\,, \qquad U_{\text{dd}}(\mathbf{r}) = \frac{\mu_0\mu_{\text{d}}^2}{4\pi}\frac{1-3\cos^2\theta}{r^3}$$

- The dimensionless dipolar Gross-Pitaevskii equation can be written as the Euler-Lagrange equation for the following Lagrangian density

$$\mathcal{L}(\psi,\psi^*) = \frac{i}{2}\left(\psi^*\dot{\psi} - \psi\dot{\psi}^*\right) + \frac{1}{2}\psi^*\nabla^2\psi - U\,|\psi|^2 - 2\pi N a_s\,|\psi|^4 - \frac{3Na_{\text{dd}}}{2}\,|\psi|^2\int d\mathbf{r}'\,\frac{1-3\cos^2\theta}{|\mathbf{r}-\mathbf{r}'|^3}\,|\psi(\mathbf{r}')|^2\,, \qquad U(\mathbf{r},t) = \frac{1}{2}\left(\gamma^2x^2 + \nu^2y^2 + \lambda^2z^2\right)$$

- We use the Gaussian ansatz with six variational parameters $\{u_i, \phi_i\}$, which are functions of time and represent the condensate widths and conjugated phases, respectively

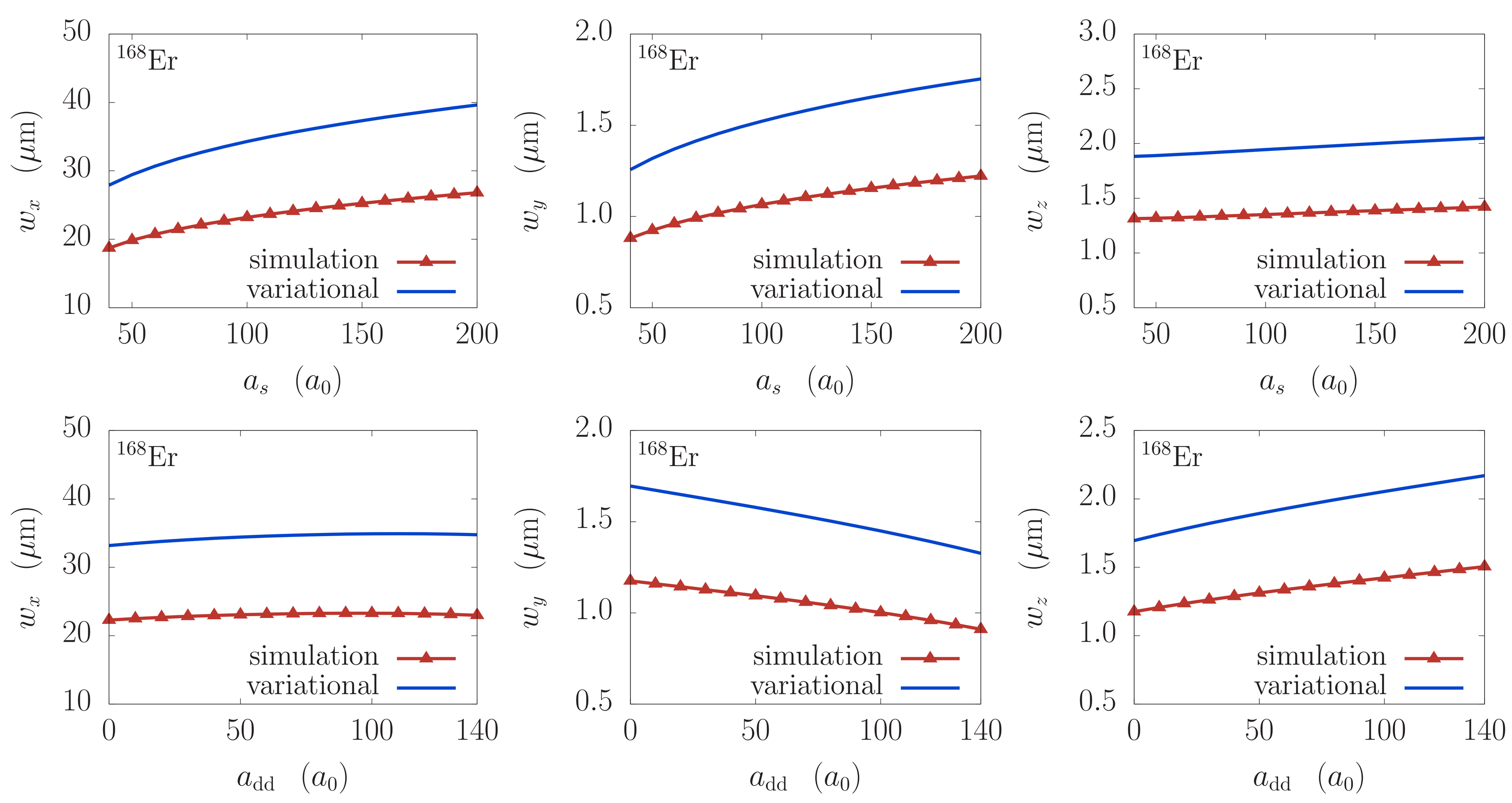
$$\psi(x,y,z,t) = \frac{1}{\pi^{3/4}\sqrt{u_xu_yu_z}}\,e^{-\frac{x^2}{2u_x^2}-\frac{y^2}{2u_y^2}-\frac{z^2}{2u_z^2}+ix^2\phi_x+iy^2\phi_y+iz^2\phi_z}$$

- The Euler-Lagrange equations for the condensate widths u_i can be expressed in terms of the anisotropy function f [4], and its partial derivatives $f_i(x_1,x_2) = \partial f(x_1,x_2)/\partial x_i$

$$L = \int d\mathbf{r}\,\mathcal{L}\,, \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0\,, \quad q_i \in \{u_x,u_y,u_z,\phi_x,\phi_y,\phi_z\} \quad \Longrightarrow \quad \begin{cases} \ddot{u}_x + \gamma^2u_x - \frac{1}{u_x^3} - \sqrt{\frac{2}{\pi}}\frac{N}{u_x^2u_yu_z}\left[a_s - a_{\text{dd}}f\left(\frac{u_x}{u_z},\frac{u_y}{u_z}\right) + a_{\text{dd}}\frac{u_x}{u_z}f_1\left(\frac{u_x}{u_z},\frac{u_y}{u_z}\right)\right] = 0 \\ \ddot{u}_y + \nu^2u_y - \frac{1}{u_y^3} - \sqrt{\frac{2}{\pi}}\frac{N}{u_xu_y^2u_z}\left[a_s - a_{\text{dd}}f\left(\frac{u_x}{u_z},\frac{u_y}{u_z}\right) + a_{\text{dd}}\frac{u_y}{u_z}f_2\left(\frac{u_x}{u_z},\frac{u_y}{u_z}\right)\right] = 0 \\ \ddot{u}_z + \lambda^2u_z - \frac{1}{u_z^3} - \sqrt{\frac{2}{\pi}}\frac{N}{u_xu_yu_z^2}\left[a_s - a_{\text{dd}}f\left(\frac{u_x}{u_z},\frac{u_y}{u_z}\right) - a_{\text{dd}}\frac{u_x}{u_z}f_1\left(\frac{u_x}{u_z},\frac{u_y}{u_z}\right) - a_{\text{dd}}\frac{u_y}{u_z}f_2\left(\frac{u_x}{u_z},\frac{u_y}{u_z}\right)\right] = 0 \end{cases}$$

Ground state

- Ground state equations are obtained by assuming $\ddot{u}_x = \ddot{u}_y = \ddot{u}_z = 0$
- Widths are defined as two times the root-mean-square of the corresponding coordinate



Collective modes

- The system is perturbed from the ground state by a small change of one of its parameters

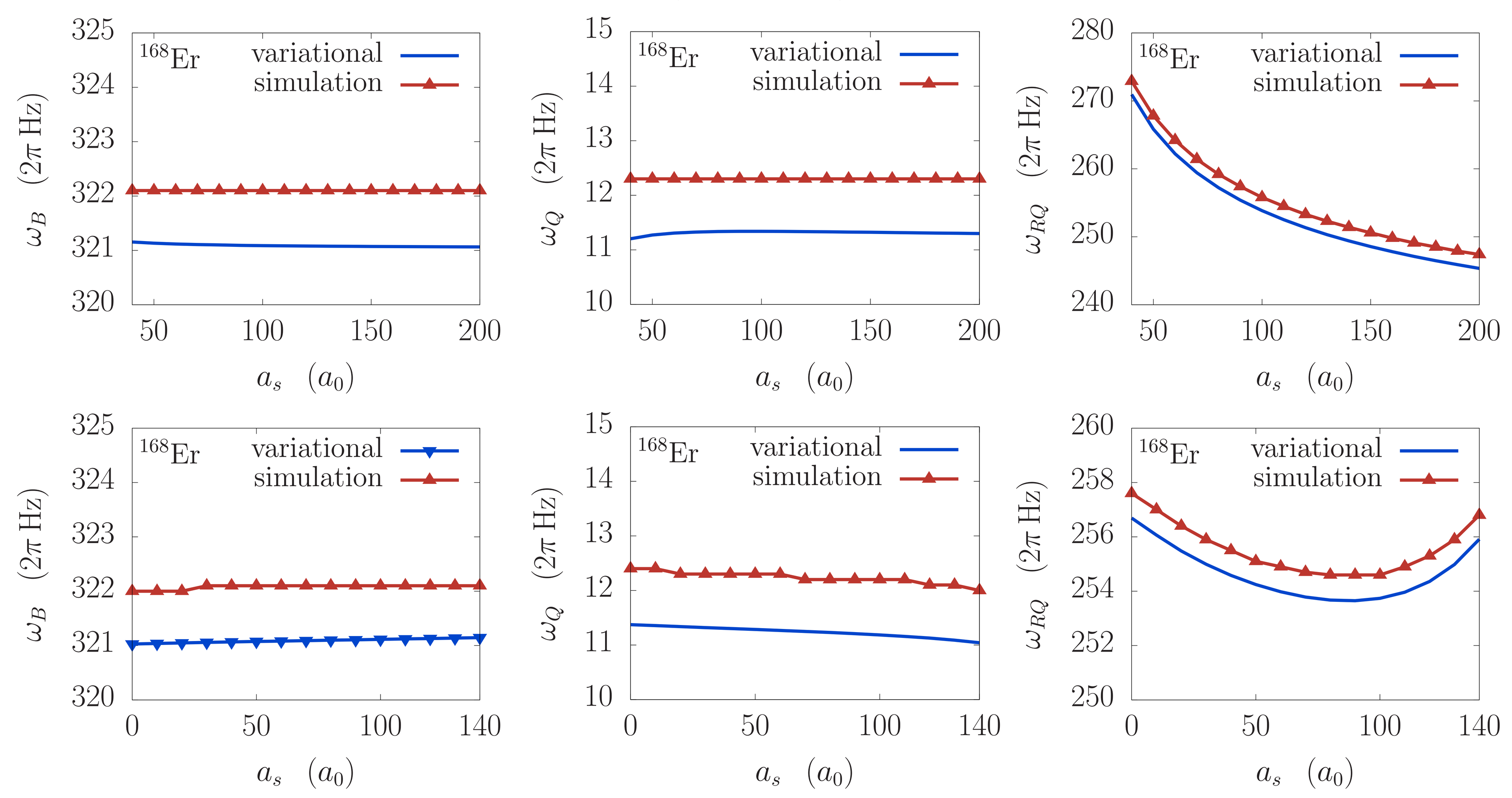
$$u_i(t) = u_{i0} + \delta u_i(t)\,, \quad i \in \{x,y,z\}$$

- Coupled system of ordinary linear differential equations of the second order

$$\delta\ddot{\mathbf{u}}(t) + M\cdot\delta\mathbf{u}(t) = 0\,, \qquad M_{ij} = -2\left.\frac{\partial^2 L(\mathbf{u})}{\partial u_i\,\partial u_j}\right|_{\mathbf{u}=\mathbf{u_0}}\,, \quad i,j \in \{x,y,z\}$$

- The collective mode frequencies are eigenvalues of the matrix M

$$\det(M - \omega^2 I) = 0$$



Conclusions

- Variational description of dipolar BECs
- Numerical solution of the dipolar Gross-Pitaevskii equation
- Good qualitative description of the system's ground state
- Excellent quantitative description of the condensate's low-lying excitation modes

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