## Experimental and theoretical study of the phase response of $M_{x}$ magnetometer to modulating transversal magnetic field

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## Introduction

The $\mathrm{M}_{\mathrm{x}}$ magnetometers in the so-called $\mathrm{M}_{\mathrm{x}}$-geometry, in which the frequency of a weak oscillating magnetic field is actively kept in resonance with the atomic spin precession at the Larmor frequency.
Investigation the phase response of a true scalar $\mathrm{M}_{\mathrm{x}}$ magnetometer to the sudden changes of transversal magnetic field [1].
Numerical and analytical modeling of a system.
Obtaining set of simple equations describing detected signal for comparison with experimental results and tracking of the phase evolution.
Presenting obtained results, and discussing which conditions lead to the signal abnormalities
Changes in the measured phase depending on the orientation of the applied modulating field.


Figure 1: $\mathrm{M}_{\mathrm{x}}$ magnetometer


Figure 2: $\mathrm{M}_{\mathrm{x}}$ magnetometer components

## Experiment

As a sensing element we employ paraffin coated cell filled with Cs.
A single light source is used for both pumping the medium and probing.
Pump light is circularly polarized.
The wave vector and RF magnetic field that drives the spin precession are at $45^{\circ}$ with the respect to the main static magnetic field $B_{0}$ as presented on Fig. 1 .


Figure 3: Experiment scheme.

The sensor head is placed inside a three layer mu-metal shielding.
Changes of the magnetometer response are detected with a lock-in amplifier, which enables us to obtain in-phase and quadrature components of the transmitted probe signal.

## Theoretical and analytical study

- Theoretical study is based on the transient Bloch equation.

$$
\dot{\vec{S}}(t)=\vec{S}(t) \times \vec{\Omega}(t)-\gamma \vec{S}(t)+\gamma_{p}(\vec{k}-\vec{S}(t))
$$

Analytically, the equation is solved in rotating frame after applying a rotating wave approximation (RWA), as it is simplest way to solve it.
The spin orientation will precess around $z$ axis thus this will be our rotation axis at frequency $\omega_{\mathrm{rf}}$ and in the same clockwise direction.
The essence of the RWA approximation is to disregard components rotating in opposite direction of the spin, as they have lower impact than co-rotating one.
In rotating frame, static magnetic field and and k vector are:

$$
\vec{\Omega}^{\mathrm{R}}=\vec{\Omega}_{\mathrm{rf}}^{\mathrm{R}}=\Omega_{\mathrm{rf}}\left(\begin{array}{c}
0 \\
\sin \alpha \\
0
\end{array}\right), \text { and } \quad \overrightarrow{\mathrm{k}}^{\mathrm{R}}=\left(\begin{array}{c}
0 \\
0 \\
\sin \alpha
\end{array}\right) .
$$

Obtained equation is free from time dependent coefficients so it can be solved analytically. We will split the solution into stationary and transient part.

$$
\vec{s}^{\mathrm{R}}(\mathrm{t})=\vec{s}^{\mathrm{RS}}+\vec{s}^{\mathrm{RT}}
$$

The spin orientation in laboratory frame $\overrightarrow{\mathrm{S}}^{\mathrm{L}}$ ( t ) is obtained after multiplying solution in rotating frame by rotation matrix.

And for phase calculation we obtain:

$$
\begin{aligned}
& \Delta \theta_{\mathrm{x}}^{\prime}(\mathrm{t})=-\frac{\pi}{2}-\arctan \left(\frac{-\mathrm{e}^{-\left(\gamma+\gamma_{p}\right) \mathrm{t}}\left(\gamma+\gamma_{\mathrm{p}}\right) \sin (\beta)}{\Omega_{\mathrm{rf}} \sin (\alpha)+2 \mathrm{e}^{-\left(\gamma+\gamma_{p}\right) \mathrm{t}}\left(\gamma+\gamma_{\mathrm{p}}\right) \sin ^{2}(\beta) \sin \left(\Omega_{\mathrm{rf}} \sin (\alpha) \mathrm{t}\right)}\right) \\
& \Delta \theta_{y}^{\prime}(t)=-\frac{\pi}{2}-0
\end{aligned}
$$



In a new coordinate system we obtain that rotation of the $\vec{B}_{0}$ around $\hat{y}$ doesn't cause any phase shift in TSM magnetometer (fast axis) while rotation around $\hat{x}$ cause transien (Fig.4).
time (s)
Figure 4: Phase evolution in case of "fast" and "slow" axis.

## Magnetic resonance

By sweeping the frequency of the rf magnetic field over Larmor frequency we observe Lorentzian resonance in amplitude R signal with maximum at $\Omega_{\mathrm{L}}=\Omega_{\mathrm{rf}}$, and a signal's phase swing from $180^{\circ}$ to $0^{\circ}$, where phase of $90^{\circ}$ corresponds to resonant conditions.
The dispersive shape of the phase curve is used to directly determine value of $|B|$ in free running magnetometers and to actively tune $\Omega_{\mathrm{rf}}$ in socalled PLL loop.


## Experimental and numerical results

The phase evolution obtained from experiment, when excitation by magnetic field is in the transverse direction, is presented in the following figures, around slow axis (Fig. 5) and fast axis (Fig. 6).

time (s)
Figure 5: Phase evolution around slow axis.

time (s)
Figure 6: Phase evolution around fast axis.

The phase response from numerical results, when $\mathrm{M}_{\mathrm{x}}$ magnetometer is in the free running mode, is presented in the following figures, Fig. 7 for slow axis and Fig. 8 fast axis.


## Conclusion

Being a true scalar magnetometer, our sensor should not experience any changes in the measured phase depending on the orientation of the applied modulating field. However, both experimental measurements and model predictions have demonstrated this is not the case

## References

[1] A. Weis, G. Bison, Z.D. Grujic, High Sensitivity Magnetometers - Magnetic Resonance Based, Atomic Magnetometers, Springer, pp 361-424 (2016).

